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On B-independence of RR Charges

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Abstract

Generalization of the recent Taylor-Polchinski argument is presented, which helps to explain quantization of RR charges in IIA-like theories in the presence of cohomologically trivial H-fields.

1 Introduction

D-branes are sources of RR fields [1]. In IIA theory a D$2p$-brane is coupled to the RR gauge fields $C^{(1)}, C^{(3)}, \ldots, C^{(2p+1)}$ (they are 1-, 3-, ... and $(2p+1)$-forms respectively). Their couplings to the brane depend on the gauge invariant field $B_D = B + F$, where $F = dA$ is the tension of $U(1)$ gauge field living on the brane, and B is the gauge 2-form, pertinent for string theory. The problem is that B is not a tension and therefore the integrals $\oint B_D^k$ over cycles in brane's world sheet are not necessarily integer-valued (only $\oint F^k$ are) and can not serve as charges [2].

In order to resolve this problem W.Taylor [3] and J.Polchinski [4] suggested to substitute the naive formula [5]

$$Q^{(1)} = \int_{M_2} B_D \tag{1}$$

for the $C^{(1)}$-charge of a topologically trivial closed D2-brane by:

$$Q^{(1)} = \int_{M_2=\partial V_3} B_D - \int_{V_3} H = \int_{M_2} F \tag{2}$$

where $M_2 = \partial V_3$ is the position of D2-brane and V_3 is any 3-volume with the boundary at M_2. If the tension $H = dB$ is an exact 3-form, $dH = 0$ (i.e. when B is well defined), this expression does not depend on the choice of V_3. The argument of [3] in favor of (2) is essentially that the bulk action mixes RR fields $C^{(2k+1)}$ with different k and one needs to diagonalize the action before defining the physical charges (as measured by remote probes). The bulk action is diagonal in terms of the tensions $G^{(2)} = dC^{(1)}$ and $G^{(4)} = dC^{(3)} - C^{(1)} \wedge H$ and the transformation from C's to G's of different degree is given by a triangular matrix. So, the diagonalization does not change the coupling to the highest RR field. In order to define effective couplings to lower RR fields, we should first integrate out all higher RR fields $C^{(2m+1)}$ with $m > k$ and then read the coupling of $C^{(2k+1)}$ to the brane. Such diagonalization procedure gives rise to non-local contributions, so that the resulting source terms are no longer concentrated on the brane world sheet. However, the non-locality disappears for constant RR fields which are used to probe the *charges*.

The purpose of the paper is to rephrase the reasoning of [3] and to generalize it to arbitrary RR fields. In the standard $d = 10$ IIA string theory the RR fields $C^{(2p+1)}$ and $C^{(7-2p)}$ are dual to each other. In this paper we ignore this restriction and consider instead a different model which preserves the RR gauge symmetries but all RR fields are independent of each other and the space-time dimension is not specified. In this case, we show that the effective coupling of the RR fields to D-branes (after integrating out higher RR fields) depends only on the tension F of the U(1) gauge field on the brane. Since our interest is in the B-dependence of the RR charges, we ignore the curvature-dependent corrections [6] and do not use the related K-theory formalism [7, 8].

The paper is organized as follows. We begin in sect.2 with the case of the D2-brane with $C^{(3)}$ and $C^{(1)}$ fields, considered in [3]. Then, after a brief discussion of the d^{-1}-operation in sect.3, we proceed in sect.4 to generic consideration of $C^{(2p+1)}$ fields (gauge odd forms) in the presence of $B^{(2k)}$ fields (gauge even forms). For cohomologically non-trivial fields H and/or topologically non-trivial branes, the gauge invariance produces constraints on the brane dynamics. They are discussed in sect.5. Sect.6 addresses the ambiguities in RR charges arising in these circumstances and their potential implications for quantum theory of branes.

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2 The case of D2-brane

The action of (the massless sector of) the IIA theory can be obtained by dimensional reduction from the $d = 11$ supergravity. In $11d$ the bosonic sector consists of the metric G_{MN} and the 3-form $\mathcal{A}_{MNP}$, subject to gauge transformations $\mathcal{A} \sim \mathcal{A} + d\sigma$ with any 2-form σ. The Lagrangian in $11d$ is:

$$L_{11} = \sqrt{G}R(G) + |d\mathcal{A}|^2 + \mathcal{A} \wedge d\mathcal{A} \wedge d\mathcal{A} \tag{3}$$

Here $|d\mathcal{A}|^2 = \sqrt{G}G^{M\tilde{M}}G^{N\tilde{N}}G^{P\tilde{P}}G^{Q\tilde{Q}}\partial_M \mathcal{A}_{NPQ}\partial_{\tilde{M}}\mathcal{A}_{\tilde{N}\tilde{P}\tilde{Q}}$, and the Chern-Simons term is independent of the metric.

After dimensional reduction to $10d$, the $11d$ bosonic fields turn into:

$$g_{\mu\nu} = G_{\mu\nu} + C^{(1)}_\mu C^{(1)}_\nu, \quad C^{(1)}_\mu = G_{\mu,11} \tag{4}$$

and

$$C^{(3)}_{\mu\nu\lambda} = \mathcal{A}_{\mu\nu\lambda}, \; B_{\mu\nu} = \mathcal{A}_{\mu\nu,11} \tag{5}$$

The $G_{11,11}$ component of the metric turns into the exponential of the dilaton field, which is irrelevant for our purposes and is omitted from all the formulas in the present paper. The gauge invariances (in addition to general coordinate transformations in $10d$) are inherited from the general coordinate boosts in the 11-th dimension ($\epsilon^{(0)}$) and from the gauge invariance of A (Λ and $\epsilon^{(2)}$),

$$\begin{gathered} B \sim B + d\Lambda \quad (\Lambda_\mu = \Lambda_{\mu,11}), \\ C^{(1)} \sim C^{(1)} + d\epsilon^{(0)}, \\ C^{(3)} \sim C^{(3)} + d\epsilon^{(2)} - d\epsilon^{(0)} \wedge B \end{gathered} \tag{6}$$

The Lagrangian in $10d$ is of the form,

$$\begin{aligned} L_{10} = \sqrt{g}R(g) + |dC^{(1)}|^2 + \left|dC^{(3)} - C^{(1)} \wedge H\right|^2 + B \wedge dC^{(3)} \wedge dC^{(3)} = \\ = \sqrt{g}R(g) + |G^{(2)}|^2 + |G^{(4)}|^2 + B \wedge G^{(4)} \wedge G^{(4)} - \\ -B \wedge B \wedge G^{(4)} \wedge G^{(2)} - \frac{1}{3} B \wedge B \wedge B \wedge G^{(2)} \wedge G^{(2)} + \text{ total derivative} \end{aligned} \tag{7}$$

Here $H = dB$, $G^{(2p+2)} = dC^{(2p+1)} - C^{(2p-1)} \wedge H$, and the total derivative appears from the transformation of the Wess-Zumino term. The bulk Lagrangian L_{10} contains terms mixing the fields $C^{(1)}$ and $C^{(3)}$. It can be diagonalized by the transformation $C \to G$. However, the inverse transformation $G \to C$ is non-local.

Introduce now even-dimensional D$2p$-branes located at M_{2p} with the world sheets W_{2p+1}. They contribute the source (Wess-Zumino) terms to the action [5],

$$\int_{W_1} C^{(1)} + \int_{W_3} (C^{(3)} + C^{(1)} \wedge B_D) + \ldots \tag{8}$$

Here $B_D = B + F$ is a gauge invariant field, which can be considered as a linear combination of the gauge field B, living in the bulk, and the tension of the $U(1)$ gauge field $F = dA$, on the brane. Under the Λ-transformation, the fields A and B transform as $A \sim A - \Lambda, B \sim B + d\Lambda$. The tension F is a closed form (satisfies the Bianchi identity), $dF = 0$, so that on the brane $dB_D = dB = H$.

The source (surface) terms (8) are gauge invariant up to total derivatives (i.e. for closed world-sheets $\partial W_{2p+1} = 0$). However, they are not expressed in terms of the $G^{(2p+2)}$-fields. If one still wants to get such an expression, a non-local formula occurs, no longer concentrated on the world-sheets. For the time being we loosely use the operation d^{-1}, its more adequate substitute will be discussed in the following section 3.

So, $C^{(1)} = d^{-1}G^{(2)}$ and

$$C^{(3)} + C^{(1)} \wedge B_D = d^{-1}G^{(4)} + C^{(1)} \wedge B_D + d^{-1}(C^{(1)} \wedge H) \tag{9}$$

In order to obtain the $C^{(1)}$ (i.e. $G^{(2)}$) *charge*, one needs to consider the coupling to D-brane of (the time-component of) the *constant* $C^{(1)}$ field (emitted or felt by a remote $C^{(1)}$-probe like a D0-brane). For constant $C^{(1)}$, however, $d^{-1}(C^{(1)} \wedge H) = -C^{(1)} \wedge B$ and we obtain for $Q^{(1)}$ exactly the formula (2). This is the argument of [3]. It deserves mentioning that the Chern-Simons term in the bulk Lagrangian vanishes for constant $C^{(1)}$.

The same argument is easily applied to the derivation of $Q^{(2p-1)}$, the charge distribution of $C^{(2p-1)}$ (provided the bulk Lagrangian for $C^{(2p+1)}$ fields is given by $|G^{(2p+2)}|^2$, as implied by duality transformations $C^{(2p+1)} \to C^{(7-2p)}$). However, in order to iterate our procedure and reach lower $Q^{(2k+1)}$ with $k < p - 1$ one can not keep $C^{(2p-1)}$ constant. This motivates a more detailed discussion of the operation d^{-1} in the next section.

3 Comment on the d^{-1} operation

Because of the nilpotency property $d^2 = 0$ of exterior derivative the d^{-1} operator is not well-defined. Its proper substitute is the K-operation (de Rham homotopy), satisfying

$$Kd + dK = 1 \tag{10}$$

As required for d^{-1}, K maps k-forms into $(k-1)$-forms. It is defined modulo d, for exact forms $K(dV) = V - d(KV) = V\mathrm{mod}(d)$. In application to integrals over topologically trivial surfaces, K can be given, for example, by the following explicit construction (known for physicists from the fixed-point gauge formalism of the early days of gauge theories, see, e.g., [9]). For a 1-form $A(x)$, the 0-form

$$KA(x) = \int_0^1 A_\mu(tx)x^\mu dt \tag{11}$$

and in general for a k-form $A(x)$

$$(KA(x))_{\mu_1\mu_2\ldots\mu_{k-1}} = \int_0^1 A_{\mu_1\ldots\mu_k}(tx)x^{\mu_k}t^{k-1}dt \tag{12}$$

or

$$\int_{S_{k-1}} KA = \int_{C(S)_k} A \tag{13}$$

where $C(S)$ is a *cone* with the base S and the vertex somewhere outside S. Of course, the K operation depends on the choice of this vertex, but this dependence results in a gauge transformation for gauge forms. The n-fold application of K builds up an n-dimensional simplex $C^n(S)$ over the surface S. For k_i-forms A_i, $\sum k_i = k$ one obtains,

$$\int_{S_{k-n}} K\left(A_{k_1}K\left(A_{k_2}K\left(\ldots KA_{k_n}\right)\right)\right) = \int_{C^n(S)_k} A_{k_1}\wedge A_{k_2}\wedge\ldots\wedge A_{k_n} \tag{14}$$

Coming back to the surface term (9), one can note that d^{-1} in this expression can be safely substituted by K, because $KdC^{(3)} \sim C_3$ modulo gauge transformation. At the same time, in variance with (9),

$$\begin{aligned}\int_{W_3}\left(C^{(3)} + C^{(1)}\wedge B_D\right) &= \int_{W_3}\left(KG^{(4)} + C^{(1)}\wedge B_D + K(C^{(1)}\wedge H)\right) = \\ &= \int_{W_3} C^{(1)}\wedge B_D + \int_{C(W_3)}\left(G^{(4)} + C^{(1)}\wedge H\right)\end{aligned} \tag{15}$$

is a well defined expression for non-constant fields $C^{(1)}$.

4 Generic RR fields

At this point we deviate from conventional IIA theory, and assume that the RR field is an odd gauge poli-form in the bulk,

$$C = \sum_{k=0} C^{(2k+1)} \tag{16}$$

with no interrelations on $C^{(2k+1)}$ imposed. The background is given by an even gauge poli-form B in the bulk,

$$B = \sum_{k=1} B^{(2k)} \tag{17}$$

and by an odd gauge poli-form A on the D-brane world-sheets,

$$A = \sum_{k=1} A^{(2k-1)}, \quad F = dA = \sum_{k=1} dA^{(2k-1)}, \quad B_D \equiv B + F \tag{18}$$

The gauge symmetries are as follows,

$$C \sim C + e^{-B}d\epsilon, \quad B \sim B + d\Lambda, \quad A \sim A - \Lambda + d\alpha, \tag{19}$$

where

$$\epsilon = \sum_{k=0} \epsilon^{(2k)}, \quad \Lambda = \sum_{k=1} \Lambda^{(2k-1)} \tag{20}$$

are even and odd poli-forms in the bulk respectively, and

$$\alpha = \sum_{k=0} \alpha^{(2k)} \tag{21}$$

is an even poli-form on the brane world sheet. The tension $H = dB$ of the B-field and the tension $G = dC - CH$ of the RR field are gauge invariant bulk fields. As before, the combination $B_D = B + F$ is a gauge invariant field on the brane world-sheet.

In analogy with the IIA theory we assume the following Lagrangian of RR fields (in neglect of possible Chern-Simons terms) in the bulk,

$$|G|^2 = |dC - CdB|^2 = \sum_i \left| dC^{(2i+1)} - C^{(2i-1)} \wedge dB \right|^2 . \tag{22}$$

Here and in what follows we often omit the sign of wedge product, meaning that contractions with the help of the metric involve either the squares $|\ |^2$ or the Hodge operation $*$. In particular $e^{-B} = e^{-\wedge B}$, $CH = C \wedge H$ etc.

The Wess-Zumino contributions of D-branes are [5]*:

$$\sum_p \oint_{W_{2p+1}} Ce^{B_D} = \sum_p \left(\sum_{i=0} \frac{1}{i!} \oint_{W_{2p+1}} C^{2p+1-2i} \wedge B_D^i \right) = \sum_{k,i} \frac{1}{i!} \oint_{W_{2k+2i+1}} C^{(2k+1)} \wedge B_D^i . \tag{23}$$

According to our general strategy we should rewrite the source term (23) in terms of G, which diagonalizes the bulk action. The resulting expression will essentially contain the K operation, but formulas for the RR *charges* are obtained at constant RR fields when the expression becomes local. Formally, in order to express C through G, one needs to solve the equation

$$G = dC - CH = (d + H)C = \left(e^{-B} de^{B}\right) \tag{24}$$

For our purposes, we need a formula with K-operation acting directly on C. Since up to a gauge transformation $C \sim K(dC)$, we write $C \sim KG - KHC$, and

$$C \sim \frac{1}{1 + KH} KG = KG - K(H \wedge KG) + K(H \wedge K(H \wedge KG)) + \dots \tag{25}$$

Since K lowers the rank of the form by one and multiplication by H raises it by three or more (note that there is no terms with $k = 0$ in (17)), this formula contains only finitely many terms for C of a given finite rank. Finally, the source term turns into:

$$\sum_p \int_{W_{2p+1}} Ce^{B_D} = \sum_p \int_{W_{2p+1}} e^{B_D} \frac{1}{1 + KH} KG \tag{26}$$

Now the entire Lagrangian is diagonalized in terms of the fields G and we can proceed to the definition of effective couplings.

Examples.

For a D2-brane we have

$$\begin{aligned} &\int_{W_3} \left(C^{(3)} + B_D^{(2)} C^{(1)}\right) = \int_{W_3} (1 + B_D + \dots)(1 - KH + \dots)(KG^{(2)} + KG^{(4)} + \dots) = \\ &= \int_{W_3} \left(KG^{(4)} + B_D KG^{(2)} - K(H(KG^{(2)}))\right) = \int_{W_3} KG^{(4)} + \int_{W_3} \left(B_D C^{(1)} + K(C^{(1)} H)\right) \end{aligned} \tag{27}$$

(we used the fact that $KG^{(2)} \sim C^{(1)}$). This reproduces eq.(9).

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*We neglect here the curvature terms, which are given by a pullback of the factor $\sqrt{\hat{A}(R)}$ in the integrand [6, 7], where the $\hat{A}$-genus is the one, appearing in index theorems, like $ind(D) = \int \hat{A}(R)\ \mathrm{tr}\ e^{-F}$.

Similarly, for a D4-brane:

$$\int_{W_5} \left(1 + B_D^{(2)} + B_D^{(4)} + \frac{1}{2}(B_D^{(2)})^2 + \ldots\right) \cdot$$
$$\cdot \left(1 - KH^{(3)} - KH^{(5)} + KH^{(3)}KH^{(3)} + \ldots\right)\left(KG^{(2)} + KG^{(4)} + KG^{(6)} + \ldots\right) =$$
$$= \int_{W_5} KG^{(6)} + \int_{W_5} \left(B_D^{(2)} KG^{(4)} - K(H^{(3)}(KG^{(4)}))\right) + \tag{28}$$
$$+ \int_{W_5} \left(\left(B_D^{(4)} + \frac{1}{2}(B_D^{(2)})^2\right) KG^{(2)} - K(H^{(5)}(KG^{(2)})) + K\left(H^{(3)}K(H^{(3)}KG^{(2)})\right)\right)$$

In order to read off the coupling of $C^{(5)}$ from (28) one notes that this field enters only through $G^{(6)}$ and the K-operation can be explicitly applied to this term: $\int_{W_5} K(dC^{(5)}) = \int_{W_5} C^{(5)}$.

The procedure to define the $C^{(3)}$-"charge" literally repeats the one for the $C^{(1)}$-charge of D2-brane. First, we assume (following [3]) that $G^{(6)}$ is already integrated out, thus one does *not* look at $C^{(3)}$ in $G^{(6)} = dC^{(5)} - C^{(3)} \wedge H^{(3)} - C^{(1)} \wedge H^{(5)}$. Then $C^{(3)}$ enters (28) only through $G^{(4)}$. Second, one should put $C^{(3)} = const$, then the K operation can be applied explicitly and give:

$$\int_{W_5} \left(B_D^{(2)} C^{(3)} - K(H^{(3)}C^{(3)})\right) \overset{C^{(3)}=const}{=} \int_{W_5} (B_D - B)C^{(3)} = \int_{W_5} C^{(3)} F \tag{29}$$

Applying the same procedure to the case of the $C^{(1)}$-charge, i.e. omitting the terms with $G^{(6)}$ and $G^{(4)}$, putting $KG^{(2)} = C^{(1)} = const$ and applying K explicitly) we obtain from (28):

$$KG^{(2)} = \int_{W_5} \left(\left(B_D^{(4)} + \frac{1}{2}(B_D^{(2)})^2\right) C^{(1)} - K(H^{(5)}C^{(1)}) + K\left(H^{(3)}K(H^{(3)}C^{(1)})\right)\right) =$$
$$\overset{C^{(1)}=const}{=} \int_{W_5} \left(\frac{1}{2}(B_D^{(2)} - B^{(2)})^2 + (B_D^{(4)} - B^{(4)})\right) C^{(1)} = \int_{W_5} \left(\frac{1}{2}(F^{(2)})^2 + F^{(4)}\right) C^{(1)} \tag{30}$$

so that for a D4-brane

$$Q^{(1)} = \int_{M_4} \left(\frac{1}{2}(F^{(2)})^2 + F^{(4)}\right) \tag{31}$$

B-independence of RR charges.

These examples illustrate the general prescription to define the effective D-brane coupling to the constant RR field $C^{(2k+1)}$: in (26) neglect all the contributions of $G^{(2m+2)}$ with $m \neq k$ and in the term with $KG^{(2k+2)}$ put $C^{(2k+1)} = const$. Then, it appears that the non-localities remaining in the K-operation can be explicitly eliminated, the contributions of B to B_D are completely canceled, and the final answer depends only on F. It deserves noting that while F itself is not gauge invariant, the integrals $\oint F^k$ over compact $2k$-cycles are.

The way to prove these claims is actually clear from above examples. It is enough to note that whenever KG in (26) is substituted by a *constant* C to obtain,

$$\frac{1}{1+KH} C \overset{C=const}{=} \sum_{j=0} (-)^j (KH)^j C = \sum_{j=0} \frac{(-)^j}{j!} B^j C = e^{-B} C. \tag{32}$$

Then, since $e^{B_D - B} = e^F$, the source term (26) becomes B-independent,

$$\sum_p \int_{W_{2p+1}} e^F C. \tag{33}$$

This formula for the effective coupling of a D-brane to the constant RR gauge fields is the main result of the paper. It shows that only the integral 2-form F contributes to the properly defined RR charges, in analogy to the claim of Taylor-Polchinski for D2-branes.

5 Implications of gauge non-invariance of the source terms

Though both the bulk and the world-sheet actions are expressed through the gauge-invariant G-fields, the non-locality of the operation K can – and does – (slightly) diminish the gauge invariance of the source (world-sheet) terms. This leads to additional constraints imposed on the brane configurations contributing to the functional integral over RR fields.

For example, the bulk action is invariant under

$$C \to C - \varepsilon\, de^{-B} \tag{34}$$

with $\varepsilon = const.$ (Indeed, $G \to d\varepsilon\, de^{-B} - \varepsilon\, de^{-B}dB$, and the first term vanishes for constant ε, while the second one vanishes because $dBdB = dB \wedge dB = 0$.) However, the boundary (source) term is not invariant:

$$\delta \oint Ce^{B_D} = \oint \varepsilon\, dBe^{B_D - B} = \oint \varepsilon\, dBe^{F} \tag{35}$$

Integration over the zero-mode ε of the full action provides the constraints,

$$\forall k \geq 0 \quad \oint_{V_{3+2k}} F^k dB = 0 \tag{36}$$

for any cycle in the D2p-brane's world-volume. In particular, for a D2-brane one gets $\oint_{W_3} H = 0$, i.e. the D2-brane can not wrap around a source of H-field (like an NS brane) in its time-evolution (such trajectories do not contribute to the functional integral).

Generalizing (34) to

$$C \to C + \varepsilon e^{-B} B^m dB$$

one obtains extra constraints:

$$\oint F^k B^m dB = 0$$

or their linear combinations $\oint F^k B_D dB_D$. At the same time, the B-independent gauge transformation

$$C \to C + \varepsilon$$

with constant ε can be easily excluded from the gauge group from the very beginning, and therefore the integrals $\oint F^k$ without B-fields need not vanish, i.e. dynamics of RR fields eliminates wrappings around "magnetic" sources of B fields, but not of F fields.

6 RR charge in cohomologically non-trivial H

When H is cohomologically nontrivial, expressions like (2), (9) and (27), in variance with the naive (1) are ill-defined. Still, it seems that they are adequate for description of the actual situation, and the ambiguities may appear to have physical significance. An ambiguity problem arises if there are non-contractable 3-cycles in the space-time with non-vanishing $\oint H$ (e.g., in Calabi-Yau compactifications) or sources of "magnetic" H-fields, where $dH \neq 0$ (e.g., the NS5-branes). Similar problems occur for topologically-nontrivial D-branes, i.e. when $M_2 \neq \partial V_3$: then (9) and (27), in variance with (2) are still applicable, but ambiguous.

In order to understand what happens in such situations, one can analyze a 1-dimensional toy-example. For a puzzle, involving realistic D2-branes, see ref.[10]. Consider the 1-dimensional Gaussian theory on a circle S_1 whose partition function is

$$\int \mathcal{D}C(x)\ \exp\left(\frac{i}{2\pi}\oint_{S_1}\left|\frac{\partial C}{\partial x} - \sigma\frac{\partial b}{\partial x}\right|^2 \ +\ C(x_1) - C(x_2)\right) \tag{37}$$

Here $C(x)$ models the $C^{(3)}$ field in the bulk, $b(x)$ and $h = \partial b/\partial x$ – the B and H fields respectively, σ plays the role of the constant $C^{(1)}$ in the bulk, and the source term $\int C^{(3)}$ (dipole charge) on the brane world-sheet is imitated by $C(x_1) - C(x_2)$. Cohomologically non-trivial h, such that $\oint_{S_1} h(x)dx \neq 0$, arises when $b(x)$ is non-periodic, $\Delta b = b(x+1) - b(x) \neq 0$, with periodic $h(x)$.

The answer for the Gaussian integral (37) (with eliminated gauge zero-mode $C(x) = const$) is

$$\exp\left[2\pi i\sigma\left(b(x_1) - b(x_2)\right)\right] \tag{38}$$

and for non-periodic $b(x)$ it is an ambiguous function of x_1 (with x_2 fixed). If there were no σ (no $C^{(1)}$ field) coupled to $h(x)$ in the bulk, one could have handled this problem by imposing a Dirac quantization condition on b: $\Delta b = integer$. However, in our model σ is a dynamical variable and, even if constant, can vary continuously. In other words, coupling to RR fields makes the phase ambiguity physically relevant. As usual in such situations, the physical state is not defined solely by the current *position* of the brane (by the points x_1 and x_2 on S_1), it also depends on its pre-history: given original state at some moment, one can obtain at another moment the physically distinct states with the same x_1 and x_2, differing by the number of times the x_1 and x_2 wrapped around the circle. The situation may be reminiscent of the theory of anyons. The state of such a system is not fully determined by current positions of the anyons, it is also labeled by an attached (invisible) braid. Moreover, the ambiguity is not automatically resolved by simply summing over all the pre-histories: dynamical constraints, discussed in the previous section lead to a superselection rule, preventing dynamical interference of topologically different pre-histories.

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